Efficient polytopal enclosures for the reachable set of nonlinear control systems

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Process monitoring and fault detection are critical activities in maintaining optimal, efficient, and above all, safe process behavior. These tasks are especially important when applied to large industrial processes, where failure can cost human lives, and the complexity of the system necessitates automation. In general, model-based techniques for fault detection can involve a nonlinear dynamic model of the state of the system. Measured system outputs can be compared to the model predictions, and excessive deviation determines a fault [4]. A critical part of this procedure is estimating the state of the system. Further, realistic state estimation should take into account uncertain parameters in the model, uncertainties in the measured inputs, and typical process disturbances. The result is then a range of process behaviors; a fault is flagged if the measured outputs deviate outside this predicted range [1].

Estimation of the range of states of the dynamic system that result from the uncertainties, otherwise known as the reachable set, is a challenging task. To make use of the nonlinear dynamic models in real time requires that efficient techniques for the estimation of the reachable set be used. The quickest techniques, based on comparison theorems, yield upper and lower bounds on each state, or an interval enclosure of the reachable set [3, 2]. However, overestimation may occur due to the wrapping of the reachable set within an interval.

This work presents a method for the construction of polytopal enclosures of the reachable set of uncertain nonlinear control systems. It is inspired by the comparison theorem results. Consequently, it has the advantage that its enclosures can be obtained very quickly, through the solution of an initial value problem in ordinary differential equations that is related to the original dynamic system. Therefore established numerical integration methods and efficient codes implementing those methods can be used. Meanwhile, the use of polytopes permits much more flexibility in obtaining a tight enclosure, and indeed significant improvements will be presented.

References

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