Before estimation of the unmeasurable states, process control would be meaningless. Consequently, observer design techniques is growing and progressing coincide with process control design, and even some control methodologies are applied in state estimation with some changes. Moving Horizon Estimation (MHE) is one of these methods which is very close to dual optimization problem of model predictive control. MHE is enough attractive that it turns to one of the most general system monitoring methods competing with extended Kalman filter for nonlinear constrained systems [1].

Although Kalman filter is an ideal state estimator for linear, unconstrained systems, it fails in nonlinear or hard constrained problems which can be found numerously in chemical engineering systems. To address these issues, the extended Kalman filter have been used in many systems in which the linearized parameters have been introduced. However, extended Kalman filter fails under the following conditions [2]:

1- At the steady state measurement, more than one value for states can be found.
2- The prior knowledge about the initial points are poor.

Note the above conditions, does not mean necessarily, the system is unobservable, but necessitate using the dynamic behavior in the state estimation. This problem, does not arise in MHE, since it is using a trajectory of measurements for estimation. In general, MHE has less restrictive assumptions in comparison than Kalman filter, but unfortunately this makes MHE method computationally very expensive.

Since MHE requires solving a nonlinear optimization recursively in each decision, it is usually limited to slow evolving or nonlinear small systems. In this work, we consider a continuous system with discrete measurements which is very common in chemical engineering systems and derive a method to speed up the optimization procedure. To achieve this goal, Carleman Linearization approach is employed to convert the nonlinear system to a linear form presentation. We assumed the uncertainties and noise value is constant in each time interval. Hence, we solved the estimator equation to obtain the error estimation. Then, the sensitivity of the estimation error and the gradient vector and Hessian matrix of the objective function are analytically derived. This information about the objective function, markedly speeds up the optimization search and reduces the errors and computational time that are usually due to numerical approximations of derivatives.

To test the proposed method, we consider a Continuous Stirred Tank Reactor (CSTR) with a nonlinear model [3]. We assumed the system is accompanied with uncertainty and uniform noise. First we show the third order Carleman linearized model can track the states of the system. Then, MHE performance is investigated while either the nonlinear model or Carleman linearized model is used in states estimation. The results show by providing the objective function gradient and Hessian simulation run decreases 10 times, while the estimation remains precise.

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